ANALYSIS OF A TEXTILE ROPE WITH ANALYTICAL MODELS

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Abstract: The stresses in the wires (fibres) forming a rope for mountaineering during sharp-edge test are analysed. A reologic approach is introduced that allows modelling a rope from single fibres since the material and cross section disposition of these are known. The limits, the difficulties, the first results and the future possibilities of this new analytical approach are presented.

1 Introduction

"The rope is the main element of the belay chain and it does not have sense to define the resistance of the other elements without considering before the study of its property."

In this sentence, drawn out from an handbook by C.I.M.T.[1] we find the nucleus of the techniques and operations developed in order to assure a safe progression in alpine activitie. The rope materials and its geometrical construction, such as the nominal disposition of fibres (wires), not only allow the rope to withstand the applied forces, but also influence the forces themselves. Such system needs therefore an approach not more static but dynamic and an accurate characterization of its mechanical properties.

A first approach can be to model a rope with a system of masses, springs, dampers whit 1 D.O.F. (Degree Of Freedom), linear whit constant coefficients, formula (1). Such a model, supported by a preventive experimental-analytical identification of its parameters [2], is able to simulate the course of the load in a rope during homologation test according to norm UNI EN 892 [3] until to the achievment of the first peak of force. Subsequently the actual rope introduces a non-linear behaviour of the mechanical properties.

$$M\ddot{x} + C\dot{x} + Kx = F \quad (1)$$

It is useful to find alternatives such as to introduce more complex systems with parameters not more constants but variable according to assigned laws. . Beyond the difficulties in the resolution of such systems, (numerical methods), the main problem is the identification of parameters. These are unknown time variable functions and are unknown. Therefore it is necessary to develop reologic models for the ropes, the most possible similar to the behaviour of the actual rope, in order to allow at least, the identification of the shapes of these functions leaving, then to an eventual numerical-experimental optimisation the searching the values for the parameters to be used.. *The aim of the present paper is to introduce this new approach*. First of all we would like to stress that this approach, considering the single fibres and not the whole rope, has the great advantage of being able to model a rope before manufacturing and testing (once the material and the geometrical construction is known). The aforementioned advantage is, up to now, only potential. Currently, because of the complexity of the analytical models, it is possible to draw only some general considerations. In particular, in this paper, we will analyze the inner stresses in a rope for alpine use during a sharp edge test through the development of two models. The first one allows the calculation of the stresses when on the rope is acting an axial force, the second one is referred to the case in which the rope axis is bent over a sheave whit assigned curvatures.

2 Short discussion about "state of the art" of rope modelling

We have to mention, first, that the present study is originated from a study carried out by author on metallic ropes [4]. Starting from this approach, some modifications have been developed to allow the application of the same models on textile ropes.

The exact calculation of the inner stresses in a metallic rope is, nowadays, a much complex exercise, both for a theoretical point of viewa and from practical one. We obtain, in fact, a system of non-linear differentials equations (dedicated for the examinatede rope) that can be solved under suitable hypotheses (sometimes very restrictive) or through numerical Still nowadays it is used to say that "designing a rope is more an art than a science". In fact a lot of acquired experience is requested and, further, safety factors (in the metallic rope) of about 10-15 are used. With regards to the modelling textile ropes problems, they increase due to hight number and very little dimensions of fibres and the great deformations of the rope during particular operating situations. Therefore there are no possibilities to use linear models (as in the metallic rope case) with a lot of substantial complications. Moreover, the friction forces (generally negligible in the short term for a well lubricated metallic rope) generate a complex dissipative phenomena and also a decreasing of the rope mechanical properties: these phenomena are very difficult to model. Due to the complexity of the problem a quite almost number of papers have been published in which the problem of analytical models has been clearly and systematically faced. As in many engineering fields, the experimental works come ahead the theory. G. A. Costello and J. W. Philips examined the highly not linear behavior (due to the variation of the pitch of helixs) of a rope stressed by a tensile force. Applying the theory of the thin and deformed beams in the space, with no friction effect between wires, a system of non linear, differentials equations has been set up that, under suitable hypotheses,

can be reduced to an algebraic equations system. This approach has been presented by the afore mentioned authors in many papers and in a book, the only one available, on ropes mechanics [5].
As far as the rope bending over an assigned radius sheave concerns, the static and fatigue behaviour is influenced by phenomena such as:

- The relative movements, in the contact points, between the wires inside of a strand, between the wires of two adjacent strands and between the wires of the rope and sheave ; it has to be noted that in the metallic rope the macro-unit used is the strand, formed by more wrapped wires, like helical spring, round to a central wire.
- The stresses variations near the contact points zone..
- The stresses due to the variation of curvature of the rope when it is bent

*<u>In this job we will analyze only the stresses from the third phenomena</u>***.** The first two ones need the development of a much more complex theory in order to model the elasto-plastic contact on no-consistent surface and in presence of friction forces [6].
The stress due to the bending in every location of the rope, bent over a sheave, is directly related to the change of local bending of the wires. They are characterized from an one initial curvature due to their helical shape (around the rectilinear axis of the rope) and assume a new curvature when the rope is bent (that is the axis of the rope is bent). From the local curvature of the wires in the two aforementioned situations it is possible to find the net variation in curvature and the stresses generated inside the wires by the bending when in the initial conditions the wires are considered as unloaded.
For how much it concerns this approach, Stein and Bert [7] present an exact mathematical metod for calculating the curvature radius of a double helix around a rectilinear axis. The curvatures calculation in the deformed rope for a single and a double helix have been carried out by Hobbs and Nabijou [8]. They explain the procedures used for the calculation of the curvatures with great details but, unfortunately, with also numerous mistakes as it can be seen from the formulas in the article. Therefore we have provided a complete rebuild of all the formulations and we obtained reasonable formulations analytically [4]. Unfortunately we obtained correct formulation only for the single curvature. Two others papers on this subject has been written by Knapp [9] and Lee [10]. They developed the same approach with the same corrections carried out by author of this paper. Unfortunately the comparison between these articles is very difficult due to the variety of adopted notations in order to identify some angles; the difficulty is not only symbolic but also connected to the choice of these angles.
In the present paper we will begin introducing these models, whit tricks dedicated to the textile ropes, in order to obtain the inner stresses in a climbing rope stressed by bending and traction in order to simulate a sharp-edge test.

3 Data
The rope model is based on the nominal geometric characteristics and the assigned section disposition.
The irregular disposition and the shape of the macro units (wicks and strands) in the rope, as well as the great variations of the shape of the section, when the rope is bending, have not been considered (due to the huge modelling difficulties): those points represent probably the most restrictive approximation of this approach. The values marked by the symbol “ * ” have been obtained directly or, when not possible, reasonably from a rope analysis. The values whit symbol “ $ “have been taken from other experimental data [11].

External rope diameter:	10.5 [mm] *
Radius of every inner wire (core):	0.0155 [mm] (0.85TEX) $
Radius of every external wire (sheath):	0.0145 [mm] (0.75TEX) $
Number central filaments (thin bundle of black wires found in the center of the rope, probably not structural):	19 *
Helix angle of the lightly twisted wires in order to form the wicks of the core:	65° (the data is a parameter that can be modified)
Helix angle of the wicks in the strands of the core:	50° *
Number of inner strands:	4 *
Number of outer strands:	9 *
Diameter of every strands:	2.56 [milimeter] (the diameter was chosen to have a total number of wires in the core near to 40000, it is however comprised in a wrap of values indicated in $)
Number of wicks in the shealth:	48 $
Helix angle of the lightly twisted wires in order to form the wicks of the sheath:	65° (the data is a parameter that can be modified)
Helix angle of the wicks of the shealth on the core of the rope:	45° $
Poisson coefficient for Nylon	0,41 ([14] pag.178)
Nylon Elastic Module	2700 [MPa]([14] pag.178)

From geometric considerations and from the reported assumptions applied to a nominal rope cross section, we obtain:

Radius of every strand of the core:	0.5940500 [mm]
Number of wires along the thickness in the radius of every strand of the core:	19
Number of wires in every wick of the core:	1066
Number of wires inside the core:	41574
Number of wires inside the sheath:	18426
Number of wires in every wick of the sheath:	409
Number of wires along the thickness in the radius of every wick of the sheath:	12

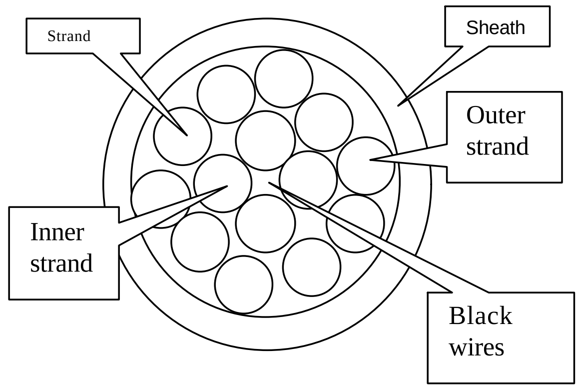

Fig.1 Cross section of a textile rope for mountaineering

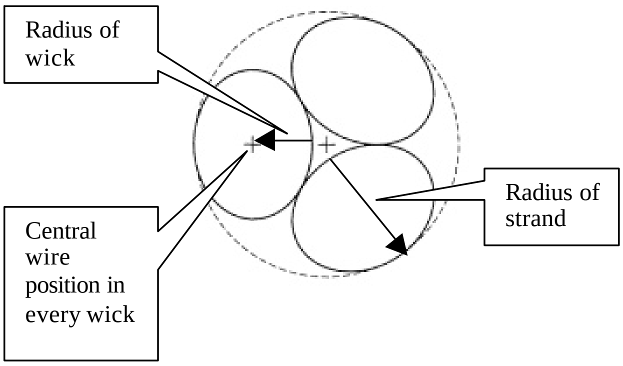

Fig.2 Cross Section of a strand

4 Axial force

4.1 Introduction to the model

Costello [5] consider the wires inside the rope like single thin beams wrapped helically around to a central wire (or core) to form a strand, which is wrapped around to a central strand to form the rope. A set of six equations (not linear) of equilibrium for every wire, considered as a deformed beam in the space, are written. . Subsequently a set of simplifications and the congruence inside the strand and the rope (the central core acts as constrain to the deformations of the other wires) has been assumed transforming the problem into an algebraic and linear one (only for metallic ropes).
We use the following general hypotheses:

- Perfectly elastic material governed by a constituent law:
-

$$s = Ee \qquad (2)$$

- Rope with nominal sizes: the rope can be deformed, contracted and rotated but no elastic-plastic deformations, both due to external force and technological process, have been considered.
- The friction between the single wires are not considered. This hypothesis is verified in case of well lubricated metallic ropes and with low force applied in which the contact stresses between the single wires are not elevated.

We observe that in the case of textile ropes these hypotheses are critical. In the case of high deformation velocity (as in the case of a dynamic fall test) the first hypothesis can be considered verified. For long times application (but also for test temperatures higher than 40° C) it would be necessary to introduce an visco-elastic behavior of the material.

4.2 The model

As far as the model concerns, the one presented in [5] and [4] with some variations illustrated afterwards In textile rope, subject to *great deformations* (near 30%), the small deformations hypothesis (generally used for metallic ropes when the force applied is very far breaking strength) is not applicable.

$$\left|\Delta a_2\right| = \left|\bar{a}_2 - a_2\right| \cancel{<<1} \qquad (3)$$

The mechanical property (stiffness) of the rope depends in fact on the pitch of the helix a_2 (and other similar angles). Due to its variation, $\bar{a}_2$, the property of the rope varies as well during the loading cycle in a non-negligible way (3).
As far as loads on the wires concerns, every wire is supposed to be provided with a punctiform area section able to support only a uniform axial force, neglecting twisting moments, shear stresses and bending moments. The simplification is correct: in fact the error obtained by the total projected wire forces on the rope axis is about 0,1 % of initial global load.

4.3 Application of the data to the model

The rope model has been built from the data of paragraph 3. The parameters used (not directly measurable or assumed in a simplified way) have been chosen in such way to analyse a Dodero dynamic fall test. They have been chosen in order to have a deformation of ~ 30% (29,69%) when a load of ~ 9KN (9002 N) is applied as medium value obtained from tests [3]. The model allows to know the axial stresses in all the wires of the rope. The maximum axial stresses of a nominal section of the rope, generated inside every wick, have been represented. Generally, in case of pure axial load, these maximum stresses are produced close to the wick centre. These stresses become lower leaving the centre.

4.4 Validation of the model

The model unfortunately cannot be validated through an analysis of absolute values. The validation consists in finding out a reasonable behaviour of the model when some data are assumed as limit data.

- F =0 the program does not supply some result due to divisions by 0.
- Equal pitch, all 90°: all the wires are equally loaded.
- Equal pitch ,all 0°: all the wires are rings around central axis. The program does not supply results.

4.5 Analysis of the stresses due to axial load

We now consider the stresses in the single wire (fibre) due to a global axial load of 9 KN. The more stressed wires are those located in the thin wrap of black filaments in the centre of the rope. Their deformation is equal to the one of the rope; a rectilinear wire is not relaxed (as for all the other wires) from helix (if a wire is helically wrapped like a spring it has more yielding when axially loaded). The stress of these wire is:

$$\sigma_{\text{axial central wire rope}} = 801 \text{ [MPa]}$$

As far as the wires concerns, it is possible to assume that in every wick there is, at least, a central wire around which the others are helically wrapped; this is true in the strands and, more in detail, in the wicks. This central wire is subject only to the curvature of the wicks in order to form the strands: the other wires are subject to the curvature to form the wicks as well. The central wire is the most stressed when an axial load, as well as with weak torsion, is applied. Since all the strands behave in the same way for a pure axial load (their position are not important), the stresses on the central wire of every wick are equal for all the wicks with the same property. There is a maximum stress value for the wicks of the core and one for those of the sheath.

$$\sigma_{\text{axial central wire core wicks}} = 423 \text{ [MPa]}$$

$$\sigma_{\text{axial central wire shealth wicks}} = 296 \text{ [MPa]}$$

The shape of the stresses distribution in the external wires of the wicks (the wires wrapped around the central wire) vary in a continuous way from a maximum, the crown of centre wire adjacent wires (shown in Fig.3), to a minimum, the more external crown. This is true for both the types of wicks, core and sheath.

$$\sigma_{\text{axial wires core wicks}} = 328 - 325 \text{ [MPa]}$$

$$\sigma_{\text{axial wires sheath wicks}} = 227 - 226 \text{ [MPa]}$$

The graphic in Fig.3 represents the maximum stress in the external wires wrapped around to the central wire within a wick.

The stresses therefore are very high, much higher than the Nylon rupture strength (not exceeding 100 MPa); two important considerations are nevertheless worthwhile making:

- These values are referred to the maximum stresses for external wire of every wick. They represent therefore an upper limit.
- The model is based on equations of equilibrium with boundary conditions of congruence between the deformations of the wires. In the model the friction due to the relative sliding of the wires is not considered

4.6 Variation of rope axial stiffness with the load

In Fig. 4 the axial load F generated from a deformation of the rope is represented. The blue curve is evaluated throughout the non-linear model developed starting from the geometry of the rope; the red curve (a straight line) is evaluated throughout the first value of pseudo-rigidity K (5). This latter curve is reported only in order to show the non linearity of the blue curve.

The blue curve seems can be interpolated as one parabola of the type:

$$F = A\left(\frac{(l_0 + dl) - l_0}{l_0}\right)^2 + B\left(\frac{(l_0 + dl) - l_0}{l_0}\right) \qquad (4)$$

where the term $\left(\frac{(l_0 + dl) - l_0}{l_0}\right)$ is the deformation of the rope. It is possible moreover to calculate the variation of the axial pseudo-stiffness K to the axial load F applied:

$$K = \frac{F}{\varepsilon} \qquad (5)$$

where å is the axial deformation of the rope (equal, for the congruence, to the axial deformation of black wires in the center of the rope, according to the congruence. In Fig. 5 it is possible to notice how the stiffness depends on the load slightly less than linearly. As it is lawful, the more the load increases, the more the rope become deformed (the helixs become more and more straight), the more the stiffness increases.

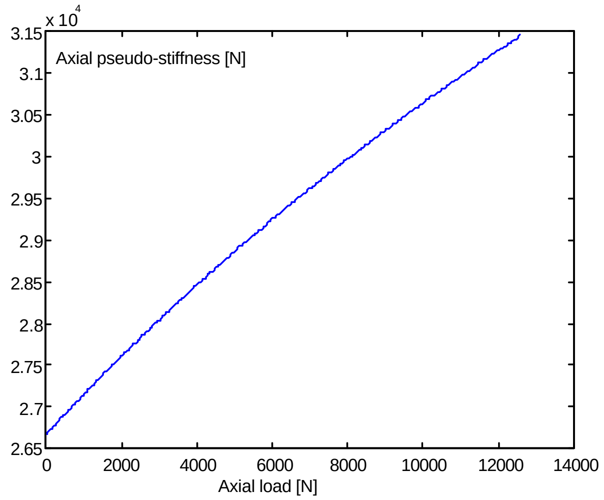

Fig. 5 Variation of the pseudo-rigidity K in function of the axial load applied to the rope

5 Bending sollicitation

5.1 Introduction to the model

This model consists in calculating for every point of all the wires in the rope, the curvatures in the undeformed and deformed condition. From the difference between the two curvatures we obtain the value to use in the stresses determination can be obtained (we assume that the formation of the rope does not generate stresses). A vector that has been assumed not to change direction represents the curvature.

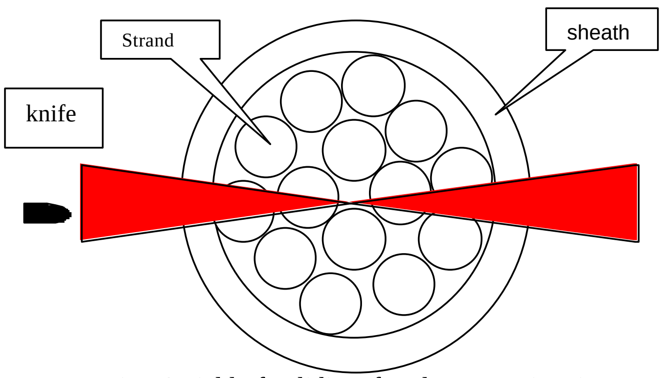

Fig. 6 Field of validity af scalar approximation

The assumption is enough correct in the metallic rope bending with low curvature but is quite critical for textile rope bent over a sharp edge. In this case the, formulation is valid only for wires disposed in a narrow arc whose axis passes through the rope centre and its contact point with the sharp edge Fig. 6.

In the following paragraphs we will use the notation adopted in [8]. It differs from that used previously for a different pich angle b instead of a (see Fig. 7). In fact in the previous axial model the pitch of a wire AB helically disposed around a direction parallel to AC axis

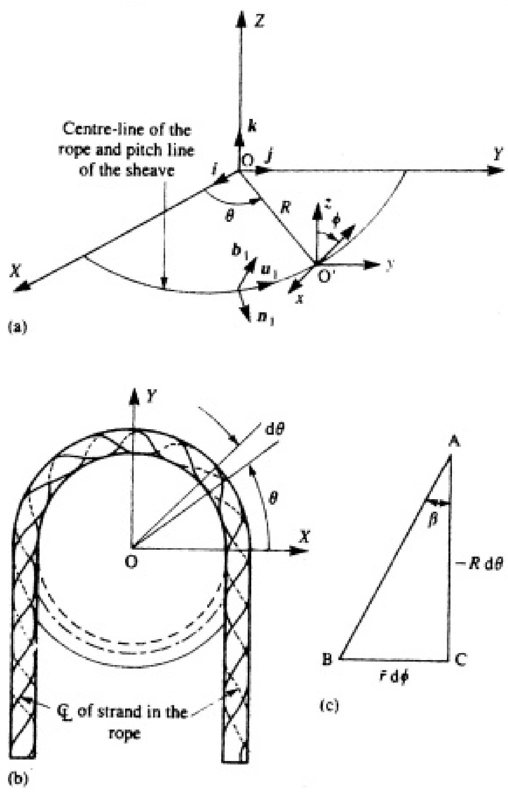

Fig. 7 Notations and sistem of reference for the model used for the calculation of the curvature

as been characterized through angle a . $A\hat{B}C$ Now instead it is characterized by the angle b $C\hat{A}B$, (see Fig. 7(c)). The model proposed allows calculating the variations of curvature only in case of single curvature that is the case of wires that are wrapped round to a rectilinear axis, when in the non-deformed condition. About the double curvature, some formulations are available in the literature but limited to simple check tests and further studies by the author of this paper have not yet reached acceptable results.

Since the wires present in the rope have been wrapped whit double and triple curvature, *the information that have been obtained, applying the model of single curvature, are to considered as upper limit of the stresses acting in every wicks* (further curvature relax bending stresses).

We emphasise that the bending model does not have constrains on forces equilibrium but only cinematic restrains, due to the curvature radius of the rope axis

5.2 The model

The rope wires axis is a three-dimensional curve in the space; every point of it is characterized by a vector position expressed in Cartesian total coordinates XYZ Fig. 7.

$$\bar{h} = X\bar{i} + Y\bar{j} + Z\bar{k} \qquad (6)$$

The curvature can be express, in vectorial shape, as [11]:

$$\frac{1}{\rho} = \frac{\left|\bar{\eta}'\times\bar{\eta}''\right|}{(\bar{\eta}'\cdot\bar{\eta}')^{3/2}} \tag{7}$$

The derivative symbol represents a differentiation regarding è that is the angle of wrapping of the rope on the cylinder (sharp edge) Fig. 7. In a single curvature case the classic formula for the single undeformed helix (with rectilinear axis) is:

$$\frac{1}{\rho_0} = \frac{sin^2\beta}{\bar{r}} \tag{8}$$

where â is the helix angle and $\bar{r}$ represents, in this notation, the distance between the axis (rectilinear) of the helix and the axis of the wire. The curvature of a helix wrapped around a circular arc (deformed case) can be calculated using previous formulation (7). The vector posizion for a single helix wrapped around a cylinder is composed
from the following members:

$$\begin{aligned} X &= (R+\bar{r}\cos\phi)\cos\theta \\ Y &= (R+\bar{r}sin\phi)sin\theta \\ Z &= \bar{r}sin\phi \end{aligned} \tag{9,10,11}$$

where ϕ it is the parametric angle along helix, â is the helix pich and R the radius of curvature of rope axis, that is the distance between the axis of cylinder on which the rope is laied down and the axis of the same rope. The Fig. 7(c) show the congruity between the helix and the arc of circumference on which the rope is laied.

$$\tan\beta = -\left(\frac{\bar{r}}{R}\right)\frac{d\phi}{d\theta} \Rightarrow \phi = -\frac{\theta R\tan\beta}{\bar{r}} + \phi_0 \tag{12}$$

where ϕ_0 represents the position of the wire, regarding the cylinder on which the rope is wrapped, in the section considered, Fig. 8.
Replacing (12) in (9-10-11) it is then possible to evaluate the derivatives, respect to è, of the members of $\bar{\eta}$. The formula (7) can be applied again in order to find the curvature. The difference, in every point, between the curvature in undeformed condition, formula (8), and deformed condition, allows to obtain the net curvature within every wire.
From an analysis of a section it is possible to have a complete picture of the stresses. The actual stresses in every point of the rope can be evaluated by analysing the variation of curvature for every wire and for a pitch. Due to the regular behaviour of the curvature in a section, the subsequent curvature variation can be seen as curvature variation within a rope for a pitch, if these are sampled in suitable way.

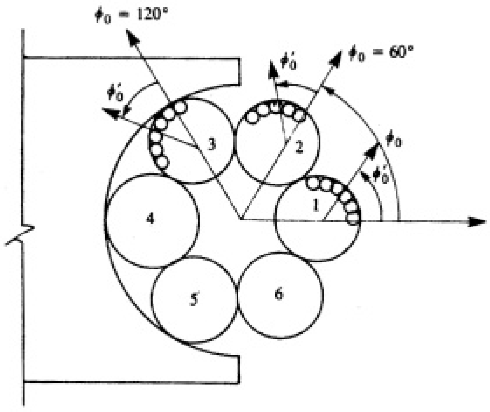

Fig.8 Positions of reference in the strands

5.3 Application of the data to the model
We built the rope model using the data presented in paragraph 3 and the parameters properly set up. The model allows the evaluation of the upper limit of the bending stresses in the wicks. The simulation of the sharp edge test with a 0.75 mm knife blade radius has been carried out. The radius of the nominal curvature of the rope axis, to be used in the numerical simulation, is:

$$R_{rope\_axis\_curvature} = R_{ropee} + R_{sharp\_edge} = 5{,}25 + 0{,}75 = 6mm \tag{13}$$

The model regards this radius as uniformly applied.

5.4 Validation of the model
The validation of the model, as before, has been carried out through the analysis of limit situations.

- R.$_{knife}$>>1 the stresses tend to 0.
- Equal pitch to 0°, all the wires (90° in the convention used for the axial model): the wire in contact with the knife introduces an exactly 0.75 mm radius of curvature. The other wires introduce curvature directly depended from their distance from the knife, and the radius of curvature of this Fig. 9.
- Equal pitch to 90°, all the wires (0° in the convention used for the axial model): the central wire is always subordinate to bending load, while the others are like rings, turn out stresses.

5.5 Analysis of the bending stresses
We can now analyse the upper limit of the stresses, Fig. 10, in the wicks due to the bending on a 0,75 mm sharp edge knife blade. We still remember the reduction of validity for the results Fig.6.

6 Conclusions

At the current state of the model prototype, it is not possible to get definitive conclusions on stresses in sharp edge test (contact with a knife blade [13]). There are still too many phenomena that are not considered in the model. It is however possible to formulate some reflections on the results carried out up to now. We note low stresses due to bending. Probably other phenomena are the principal responsible for breaking. Moreover the axial force (during Dodero fall test) generates an enough uniform and high stress level (actually we do not know exactly the absolute stresses but we know that the rope can support this load, during Dodero tests, for a limited number of times, being very closed to the rupture strength) and so a light reduction of rope section can generate a catastrophic process.

7 Future developments

The future developments of this rheological approach are remarkable but there are some difficulties. Sure the final goal of a such method is the analytical design of a rope but, currently, we are far from being able to model exactly one rope composed from 60000 fibres disposed on multiple curvatures and with huge possibilities for fibres layout. Following a step-by-step approach, tests have to be made in order to validate the model evaluating the stiffness variation due to the rope deformation; suitable experimental parameters have to be identified

For this purpose, dynamic tests suitably recorded on the Dodero machine, can be carried out with different masses.

It would be possible to implement the model with a viscous-elastic material constituent law as well [14].

Further parameters could be measured during slow traction tests according to an analytic-experimental optimisation.

In both of tests it is possible to use the new equipments available in University of Padova [15].

As far as the analytical development concerns, an exact theory for wires analysis with more than double curvature in requested as well as a completely vectorial bending theory, direct consequence of the analytical approach.

These evolutions, conceptually easy, are practically very complex.

Dissipative phenomena deserve a particular attention. It seems, from the results, that its contribution is fundamental for realistic understanding the stress distribution within a rope. One small part of it could be already available from the viscous-elastic constituent law but the great part, due to the sliding of adjacent wires, still needs an efficient model. The difficulties in this case are enormous since would be necessary to calculate, beyond the relative sliding, the contact forces point by point.

Acknowledgments

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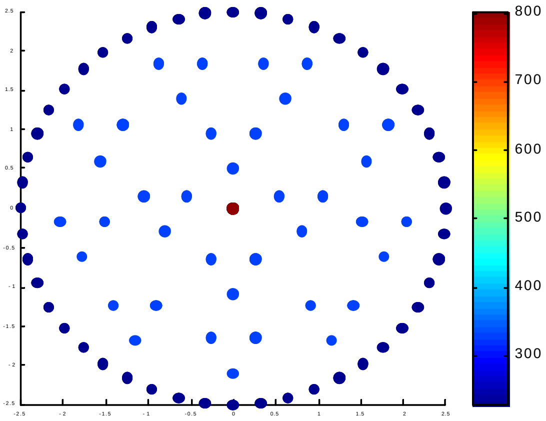

Fig. 3 Maximum axial stresses (9 KN external load) in the external wires of every wick [MPa]

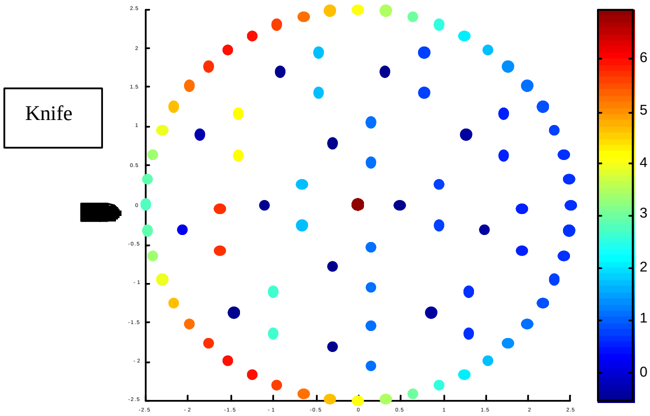

Fig. 10 Simulation of a bending load (due to $R_{curv.}$ = 0.75 mm), maximum stresses of the central wires of every wick [MPa]

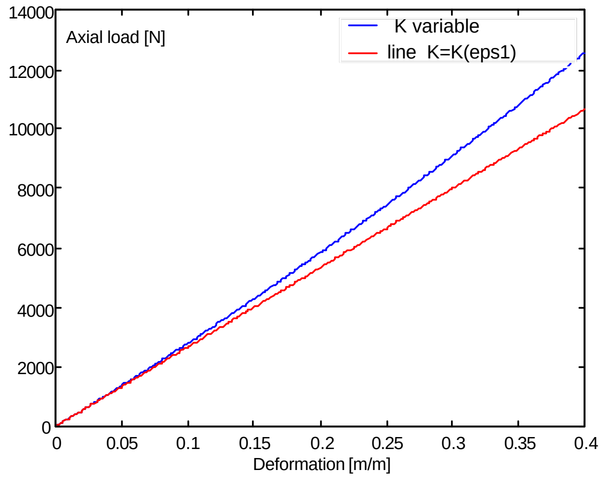

Fig.4 Course of the axial load with the deformations

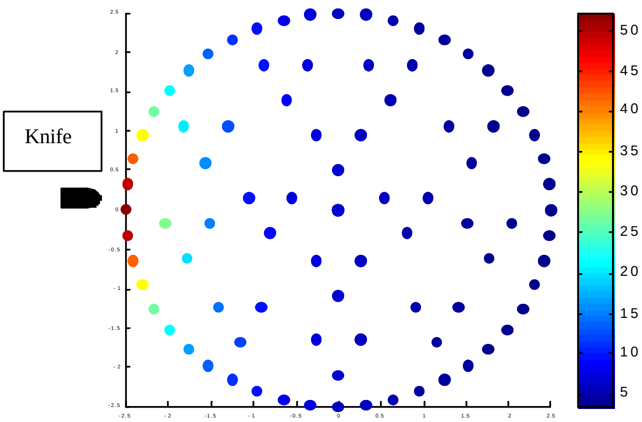

Fig. 9 Simulation of bending load when all the pitches are placed to 0°: maximums axial stressses [MPa]