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A constitutive equation for the behaviour of a mountaineering rope under stretching during a climber's fall

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Abstract

Leader fall is the main concern in climbing activities and the appropriate answer to the problem is the rope.

An accurate numerical simulation of the consequences of a fall on the “safety chain” (assembly of rope, fixed points, karabiners, cords and other climbing gears) is essential for understanding the events and conceiving safety methods. It also reduces the times and costs involved in experimental assessments. In this context a sound knowledge of the rope properties is essential.

The analytical model of the rope behaviour during stretching should be slim enough to reduce computing time, but still be able to describe the rope behaviour in each segment of the “safety chain”.

The present paper proposes a model to describe the rope behaviour when stretched, representing the force as a function of strain and strain rate. Model parameters have been calibrated on experimental data measured from a specific mountaineering commercial rope. The same procedure can be applied for different mountaineering ropes present on the market.

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1. Introduction

In climbing and mountaineering activities, the most critical piece of equipment in terms of safety is the so called “dynamic rope”, able to stretch under dynamic load and therefore capable of absorbing energy, thus reducing the shock forces acting on the human body and on the “safety chain” (assembly of rope, fixed points, karabiners, cords and other climbing gears). Mountaineering ropes are considered personal protective equipment (PPE) and submitted to the EN 892 standard [1].

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In the analysis of the loads occurring on the elements of the "safety chain" and on the human body, the use of mathematical models becomes an useful tool for the optimization of the equipment assemblies and safety methods. This reduces time and costs in experimental testing [2] and improves the awareness of the phenomena.

The setup of such models requires an analytical formulation of the rope behaviour in terms of a relation between load, strain and, also, strain rate. A great amount of work has been devoted to this subject during recent years. A growing amount of evidences is available in literature [3] [4] [5] [6].

Mountaineering ropes have a quite complex structure due to the "kernel-mantel"-like construction; accordingly models describing all the physical mechanisms that take place during rope stretching are indeed very sophisticated. The reasons for that are the visco-elasto plastic behaviour of the polyamide PA basic material, the friction between yarns, strands and other constitutive parts of the rope and the time dependent strain recovery during a possible unload lapse of time between two adjacent loading cycles. All these characteristics can change from rope to rope according to the manufacturing design.

An overall rope model (able to describe the behaviour up to the yarn level) is by far too complex, considering that a rope is composed of 60.00 yarns. For the aim of this research a simple model is required. Nevertheless, even in its simplicity, the constitutive equation should cover all the main features of the rope including non-linear behaviour, energy dissipation during the load-unload cycles and time-dependent strain recovery.

In the past, several models of various degrees of simplicity were proposed to describe the mechanical characteristics of elastic components such as the Maxell or Kelvin Voigt ones or a combination of them. To our knowledge, none of these models comprehensively mimics the mountaineering rope's behaviour.

For this purpose, a modified version of the non-linear model proposed by Hunt and Crossley [7], well known in the field of impact studies, is proposed in the present study. This model defines the force generated in the rope as the sum of two terms: the first describes the dependence of the force on the rope strain by a nonlinear formulation; the second one describes the dependence of the force on the rope strain as well as on its strain rate so enabling the energy dissipation.

Nomenclature

t	time
x	mass displacement
x'	mass speed
ϵ	rope strain
ϵ'	rope strain rate
F	force in the rope
L	specimen length
H	free fall height
ff	fall factor H/L
ci	constitutive equation coefficients
<i>subscript</i>	
s	refers to the mass movement start
t	refers to the rope stretching start
res	stands for "residual" (referred to strain after distressing)
unload	stands for an instant in which load disappears



FIG. 1 The experimental apparatus (schematic) and a detail of the rope-mass connection (two clamps)

2. The reference experimental data

The definition of the parameters of the rope constitutive equation was obtained by a best fitting procedure starting from 4 experimental data sets; carried on with 4 specimens cut from the same rope. A mass, able to fall vertically and freely with no friction between two guiding columns, is arrested by a piece of rope tied to the mass on one side and fixed to an anchor point on the other side. In these circumstances the rope practically absorbs the entire fall energy. The experimental data refer to a commercial mountaineering rope.

During the test, the mass displacement and the generated force were recorded. The sampling frequency was 1 kHz; four different experimental situations were considered, i.e. four different fall factors: $ff=2$, $ff=1,5$, $ff=1$, $ff=0.5$.

The mass speed was calculated by numerical derivation and subsequent filtering of the mass displacement measured by laser as a function of time. In order to reduce energy dispersions from the rope to a minimum, the fixed constrain was obtained by a mechanical clamp, thus avoiding energy absorption by knots. In Fig.1 the scheme and a detail of the experimental apparatus is shown. Fig. 2 depicts mass displacement and speed as a function of time referred to $ff=1.5$; the force is represented as well. The classical diagram showing the force and the strain rate as function of strain is represented in Fig. 3. The data are relative to a 2.38 meter long specimen obtained from a modern mountaineering rope; the fall height was 3.52 meters.

The following equations define strain and the strain rate:

$$\varepsilon = (x - x_t) / L \quad (1)$$

$$\dot{\varepsilon} = \dot{x} / L \quad (2)$$

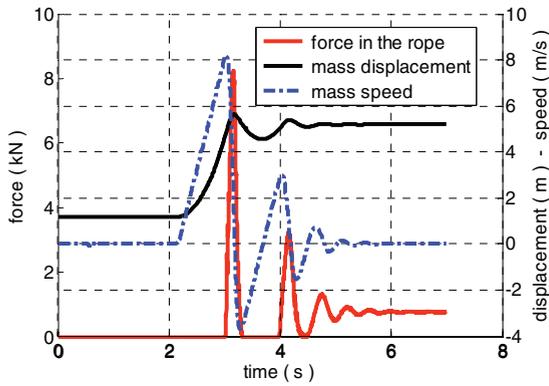


FIG. 2 Force, displacement and speed versus time

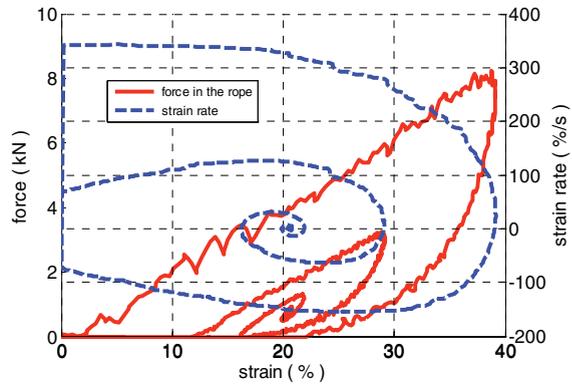


FIG. 3 Force and strain rate versus strain

3. The constitutive equation

The equation describing the force in the rope is of the type $F = f(\epsilon, \dot{\epsilon}, \epsilon_{res})$, composed by two parts: the first represents the contribution due to the strain alone; the second describes the additional effects of the strain rate. The residual strain is involved as well. This formulation enables a situation in which a lack of force is possible even with a non-zero strain, typical of a complete distressed phase between two subsequent load sequences as shown in Fig. 2 and FIG. 3. The mass rebound sequences are well deducible from the mass displacement shown in Fig. 2.

The force is represented as follows:

$$F_{load} = c_1 (\epsilon - \epsilon_{res})^{c_2} + c_3 (\epsilon - \epsilon_{res})^{c_4} \epsilon^{(1-c_5 \dot{\epsilon})} \tag{3}$$

$$F_{unload} = c_1 (\epsilon - \epsilon_{res})^{c_2} + c_6 (\epsilon - \epsilon_{res})^{c_7} sign(\dot{\epsilon}) |\epsilon|^{(1-c_8 \dot{\epsilon})} \tag{4}$$

The equations take into account the time dependent residual strain (ϵ_{res}) that was calculated at each

time-step by means of the strain recovery rate ($\dot{\epsilon}_{res}$) (evaluated as a mean of the measured ones during unload phases) and considered linear in time.

The current residual strain is represented by:

$$\epsilon_{res} = \dot{\epsilon}_{res} (t - t_{unload}) \tag{5}$$

The eight coefficients c_i comparing in (3) and (4) were defined by means of a best fitting process minimizing the sum of the differences between the measured and calculated forces at each time step. The calculated force, according to (3) and (4), were obtained using experimental strain and strain rates during the entire process. The mathematical algorithm of the simplex search method [8] was used.

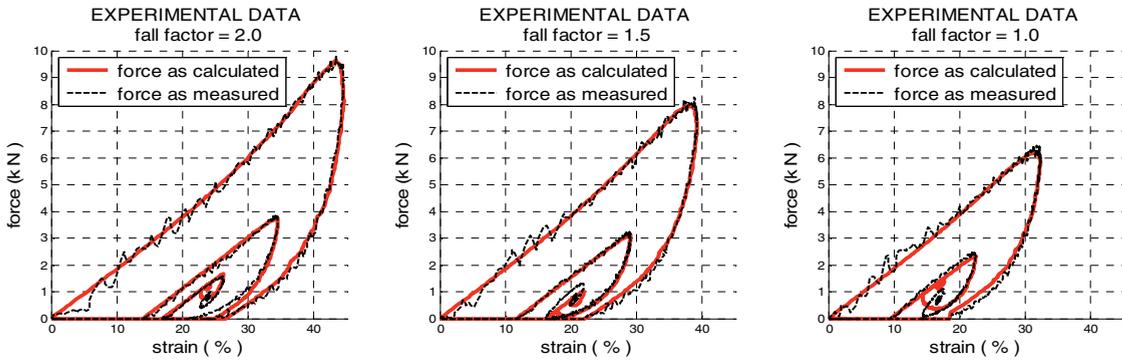


FIG. 4 Comparison between force (as measured and calculated) versus strain for three fall factors

The sum procedure was further extended to the four experimental sets of data corresponding to ff 2, 1.5, 1 and 0.5. The optimization was performed subdividing the load history into three steps, corresponding to the three sequences “load-unload” deducible from Fig. 2 and Fig. 3.

A comparison between the experimental data and the calculated force is shown in Fig. 4. Note the good correspondence between the two slopes in the outer loop, except in areas with low force value, mostly in the unloading phases. As consequence, accounting for the areas involved in each loop, the dissipated energy as calculated is greater than the measured one. A mild discrepancy is present in the inner load-unload loops as well.

4. Comparison between the reference experimental data and the results of a fall model using the rope constitutive equation

A numerical simulation of the experimental fall was performed using Simulink/Matlab language; the force generated in the rope was calculated according to the constitutive equations (3) and (4). The parameters of the experimental fall are recalled here:

- Falling mass M 80 kg
- Rope length L 2.384 m
- Free vertical fall H 3.521 m
- Fall factor $ff=H/L = \sim 1.5$

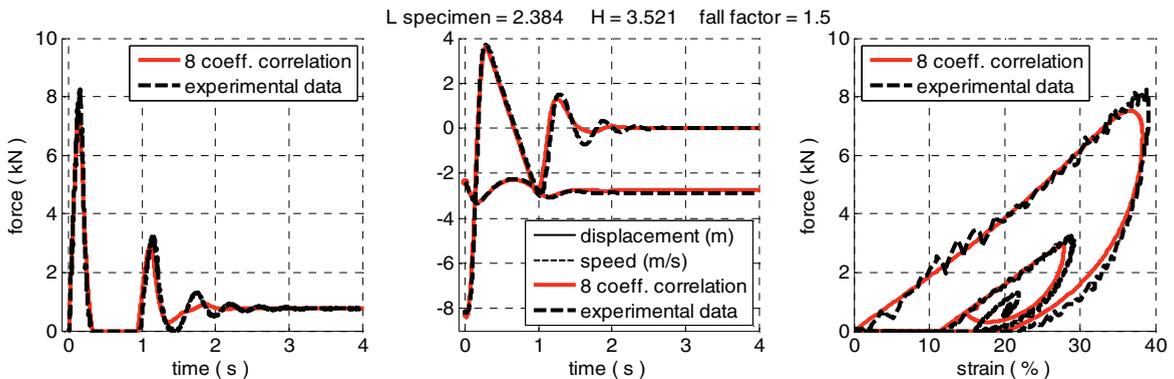


FIG. 5 Comparison between measured and calculated data

The comparison between the calculated data and the experimental ones are reported in Fig. 5. It is worthwhile noting the good agreement between calculated and measured data, particularly for the outer load loop; the inner loops are affected by the strain recovery rate that has been assumed as the mean of the measured values; this assumption seems to be slightly poor and will be improved in the future. A mild discrepancy between the maximum force measured and the calculated one is evident, this could depend on the quality of the correspondence between the experimental apparatus and the overall mathematical model. However, the model is able to reliably predict the first peak zone of the fall (and the second as well): this is the most critical zone covering the highest loads in the process. Therefore the model could be used in further analyses to simulate and eventually optimize the use and effectiveness of the mountaineering gears involved in the “safety chain”.

Conclusions

A mathematical model describing the behaviour of a rope under loading and unloading cycles during a climbing fall is presented. The model defines the force generated in the rope as a function of the strain and strain rate as well as of the strain recovery rate, that is $F=f(\varepsilon, \dot{\varepsilon}, \varepsilon_{res})$. The proposed equation fits well with the experimental data; still, the first load/unload loop is better represented than the inner ones. This model will probably be a satisfactory tool to simulate the behaviour of a rope in climbing falls, including complex situations whereby several “runners” are involved in the safety chain and the rope is not fixed at an anchor, but sliding in a braking device, according to the "dynamic belay" technique.

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